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One might say that physicists study the symmetry of nature, while mathematicians study the nature of symmetry.

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@GWOMaths Observing symmetry in nature, such as noting the similarity between the symmetries of a snowflake and a hexagon, is readily comprehensible. What does it mean then to study "the nature of symmetry"?

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@GWOMaths Mathematicians define a "group", G, as a set of elements {a,b,c, ...} with a binary operation ■ and a distinguished element e (the identity of G) satisfying these specific properties:

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@GWOMaths For all a, b, c in G
1) Closure: a■b is in G.
2) Associativity: (a■b)■c = a■(b■c)
3) Identity: e■a = a■e = a.
4) Inverse: There exists an element a* such that a*■a = a■a* = e.

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@GWOMaths Just as functions on the integers, rationals, or reals are defined as mappings, mathematicians define a *group morphism* μ as a mapping from one group to another that preserves the group structure:

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@GWOMaths For groups G = [{a,b,c, ...}, ■, e] and H = [{α,β,γ, ...}, ■, ε]
then μ: G ■ H is a "group morphism" if for all elements of G:
μ(a■b) = μ(a)■μ(b). Note that for all a:
μ(a)■ε = μ(a) = μ(a■e) = μ(a)■μ(e)
and hence μ(e)=ε; Similarly it can be shown that

μ(a*) = μ(a)*.

@GWOMaths Thus the definition of such a *group morphism* preserves the group structure. When such a *morphism* is both *onto* (ie every element of H is mapped to by one or more elements of G) and *one-to-one* (only one element of G maps to each element in H) it is termed an *isomorphism*.

@GWOMaths For mathematical purposes, when there exists an *isomorphism* between two groups G = [{a,b,c, ...}, ■, e] and H = [{α,β,γ, ...}, ■, ε] then G and H are termed *isomorphic*, or *the same up to isomorphism*.

@GWOMaths Now all the finite groups can be classified in terms of various internal structures, first collecting those which are *the same up to isomorphism* and then collecting families with similar internal structure.

@GWOMaths When all the families of groups - Cyclic, Alternating, and assorted Lie Group Types - have been defined, there are remaining 26 groups that don't fit anywhere: the *sporadic groups*.

@GWOMaths Of these 26 *sporadic groups*, two stand out from the others in terms of their size:

- the *Baby Monster*, *B*, of size
$2^{■1} \cdot 3^{13} \cdot 5^{■} \cdot 7^{2} \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 31 \cdot 47$; and

@GWOMaths - the *(Fischer–Griess) Monster), *M*, of size
$2^{■■} \cdot 3^{2■} \cdot 5^{■} \cdot 7^{■} \cdot 11^{2} \cdot 13^{3} \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31 \cdot 41 \cdot 47 \cdot 59 \cdot 71$.

Counting up the number of distinct primes in that last number gives us 15.

@GWOMaths Therefore today's answer is that:

The number of distinct prime factors
in the size, n, of the *Monster Group* M
is

15.

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