Twitter Thread by Algebra Etc.

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@AlgebraFact

How to mentally compute the cube root of the cube of a two digit number.

Thread (1/10)

In base 10, the last digits of the cubes of digits are distinct.

If d = 0, 1, 4, 5, 6, or 9, then d^3 ends in d.

If d = 2, 3, 7, or 8, then d^3 ends in 10-d.

(2/10)

So if you're given the cube of an integer, you can figure out the last digit of the cube root.

For example, if 50653 is the cube of an integer, it's the cube of a number ending in 7.

(3/10)

Let n = 10a + b be a two-digit number, with a and b single digit numbers.

Then 10a <= n <= 10(a+1), and so n^3 is between (10a)^3 and (10(a+1))^3, i.e. between 1000 a^3 and 1000 (a+1)^3.

(4/10)

So suppose x is the cube of a two-digit number n = 10a + b.

Chop off the last three digits of x.

Then a, the first digit of n, is the largest number whose cube is no greater than what's left of x.

(5/10)

This means that if you know the cubes of the digits, you can find the first digit of the cube of a two-digit number.

(6/10)

For example, suppose we know 50653 is a cube.

Lopping off the last three digits gives 50.

3^3 < 50 < 4^3, so the first digit of the cube root of 50653 is 3.

(7/10)

At the beginning of this thread we showed how to find the last digit of the cube root, and now we've shown how to find the first digit.

(8/10)

Another example. Suppose you know that 438976 is a cube.

The last digit of the cube root must be 6.

438 is between 7^3 and 8^3 and so the first digit of the cube root must be 7.

438976 = 76^3.

(9/10)

Final example. Suppose you know x = 103823 is a cube, x = n^3.

Then the last digit of n is 7 because 7^3 ends in 3.

103 is between 4^3 = 64 and 5^3 = 125, so the first digit of n is 4.

n = 47.

(10/10)