Twitter Thread by 10-K Diver

10-K Diver
@10kdiver

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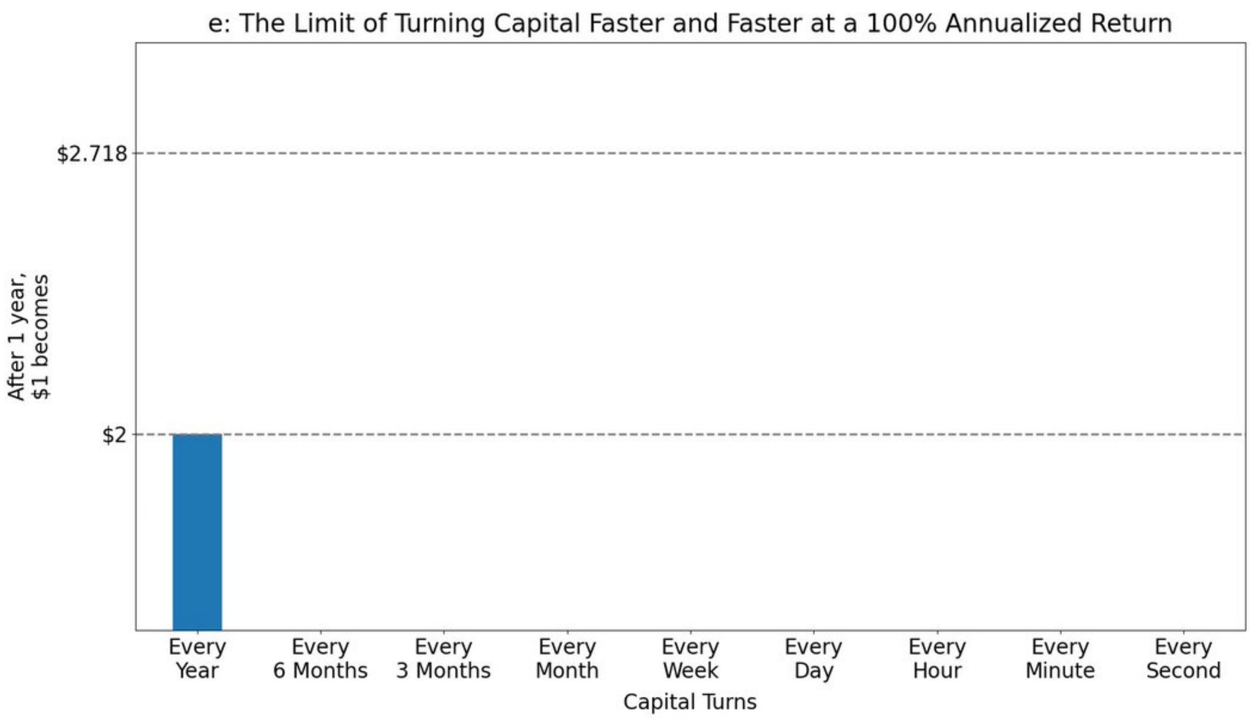
Get a cup of coffee.

In this thread, I'll walk you through the benefits of turning capital quickly.

The math behind turning capital is beautiful. It leads us to the number "e", which plays a vital role in so many different fields -- from astrophysics to biology.



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Imagine we have $1M.

Also, we know an extraordinarily generous bank where we can deposit this $1M.

This bank will pay us interest. And not a paltry 1% or 2%, but a hefty *100%* per year!

(I know, I know. But humor me, will you?)

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So, if we deposit our $1M at this bank, it will earn that 100% interest and become $2M in 1 years' time.

But that's *only* if the interest is *compounded annually*.

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What if the interest is *not* compounded annually?

For example, suppose it's compounded half yearly?

"Compounded half yearly" means: instead of paying us 100% interest *once* a year, the bank pays us 50% interest *twice* a year (ie, once every 6 months).

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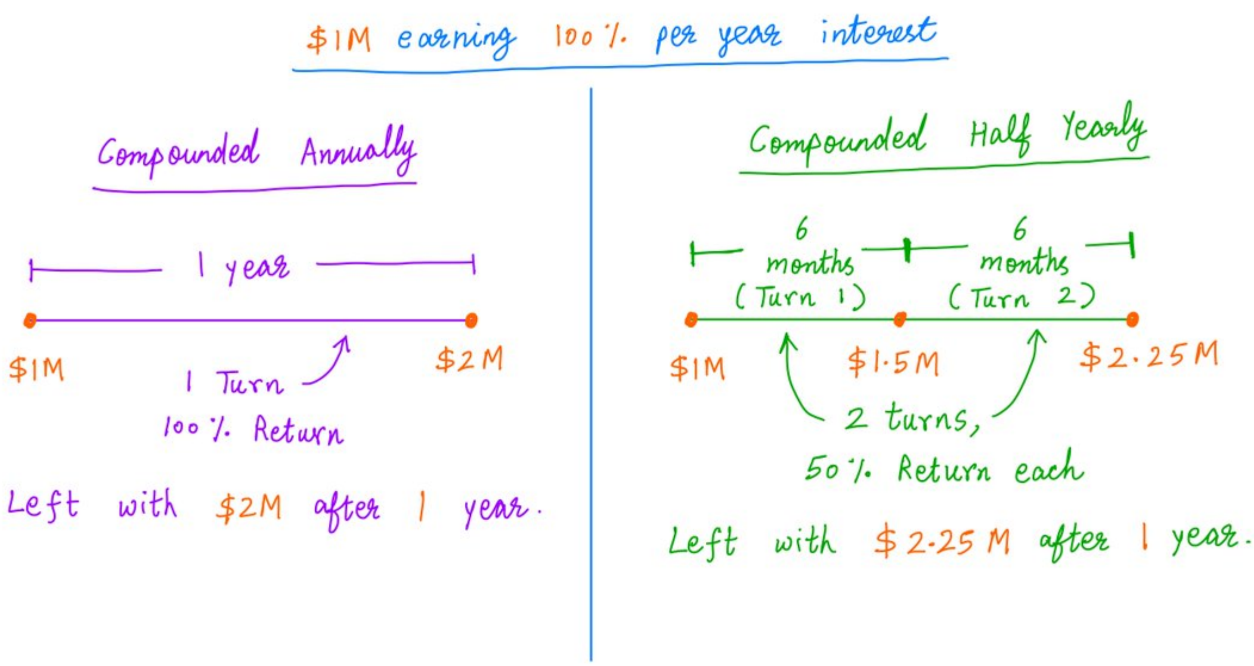
That is, we get to "turn" our money twice -- each time earning 50% on it.

Here's a quick picture of how that looks.

The first 6 months will turn $1M into $1.5M (ie, +50%). And the second 6 months will turn this $1.5M into $2.25M (another +50%).



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So, compounding half yearly works out *better* for us than compounding annually.

Half yearly means we end up with $2.25M after 1 year.

Annually means we end up with only $2M in the same 1 year.

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This is a *fundamental* fact about turning capital.

In general, the more quickly we can turn our capital, the higher our return will be.

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For example, the picture below shows how much we end up with if we turn our capital even faster.

As the table shows, *quarterly* is even better than *half yearly*.

Monthly is even better than *quarterly*.

Daily is even better than *monthly*.

And so on.

	$1M earning 100% per year interest				
	Compounded				
	Annually	Half Yearly	Quarterly	Monthly	Daily
# Turns Per Year	1	2	4	12	365
Return Per Turn	100%	50%	25%	8.33%	0.27%
After 1 Year, $1M becomes	$2M	$2.25M	$2.44M	$2.61M	$2.71M

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But there also seems to be a "law of diminishing returns" at work.

For example, doubling our turns from annually to half yearly netted us $250K ($2M -> $2.25M).

But doubling our turns *again*, going from half yearly to quarterly, only netted us ~$191K ($2.25M -> $2.44M).

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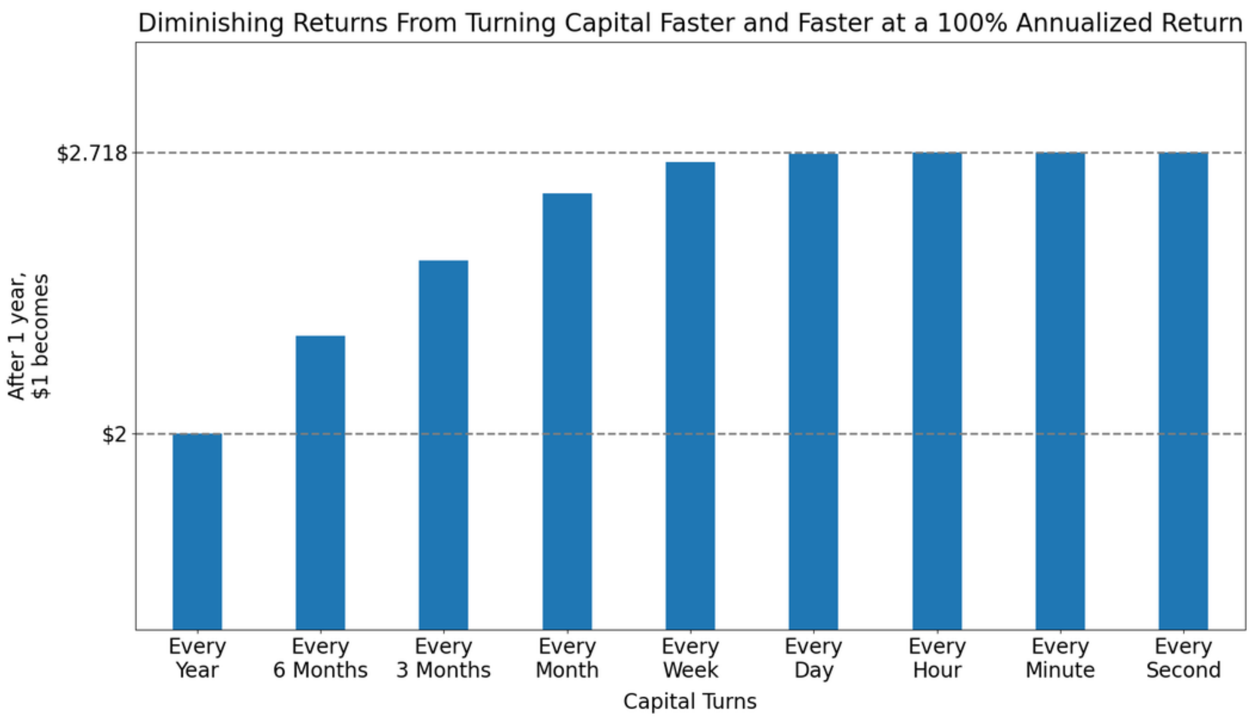
This also is a fundamental property of capital turns.

As we crank up our turns more and more, the incremental benefit we get becomes less and less.

We still get a positive benefit each time. But the benefit gets smaller as we go:



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If we dig a little deeper, we can come up with a formula.

Suppose our capital turns N times per year.

Our formula then predicts how much each $1 of our money will become after 1 year -- as a function of N.

Here's the formula and its simple derivation:

$1 earning 100% per year interest, turning N times per year

$$\#\text{ turns per year} = N$$

$$\text{Therefore, Return Per Turn} = \left(\frac{100}{N}\right)\%$$

$$\text{So, in 1 year, each \$1 becomes: } \$1 * \left(1 + \frac{\frac{100}{N}}{100}\right)^N$$

$$= \boxed{\left(1 + \frac{1}{N}\right)^N \text{ dollars}} \quad //$$

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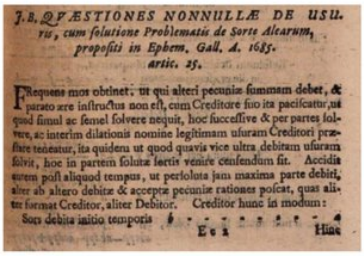
Jacob Bernoulli was one of the first mathematicians to study this formula in the context of capital turns and compounding money.

Bernoulli's work dates all the way back to 1690!

That's when he published a paper about this formula in the journal Acta Eruditorum:



J.B. QUÆSTIONES NONNULLÆ DE USU-ris, cum ſolutione Problematis de Sorte Alearum, propoſiti in Ephem. Gall. A. 1685. artic. 25.

FRequens mos obtinet, ut qui alteri pecuniæ ſummam debet, & parato ære inſtructus non eſt, cum Creditore ſuo ita paciſcatur, ut quod ſimul ac ſemel ſolvere nequit, hoc ſucceſſive & per partes ſolvere, ac interim dilationis nomine legitimam uſuram Creditori præſtare teneatur, ita quidem ut quod quavis vice ultra debitam uſuram ſolvit, hoc in partem ſolutæ ſortis venire cenſendum ſit. Accidit autem poſt aliquod tempus, ut perſoluta jam maxima parte debiti, alter ab altero debitæ & acceptæ pecuniæ rationes poſcat, quas aliter format Creditor, aliter Debitor. Creditor hunc in modum:

Sors debita initio temporis

E e 2 Hinc

> An account starts with $1.00 and pays 100 percent interest per year. If the interest is credited once, at the end of the year, the value of the account at year-end will be $2.00. What happens if the interest is computed and credited more frequently during the year?

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In the paper, Bernoulli proved that the formula *converged* to a limit.

Here's what that means:

As we keep increasing the number of turns per year N, the number (1 + 1/N)^N (which is what each dollar of our money becomes after 1 year) bumps up against a hard ceiling.

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Bernoulli proved that this ceiling had to lie somewhere between 2.5 and 3.

That is, *no matter how fast* we turn our capital (even if we turn it every nanosecond, or a billion times faster than that), we cannot grow $1 into $3 or more in 1 year. The ceiling prevents us.

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We now know that the ceiling is ~$2.718 -- between $2.5 and $3 as Bernoulli knew it had to be.

If we turn our capital *infinitely* often, we will grow $1 into roughly $2.718 in 1 year. Not more.

There's a strong law of diminishing returns at play.

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This magic number (2.718281828...) is called "e".

e has a special place in mathematics.

It's one of the most important universal constants. And it turns up in all sorts of surprising places -- from probability distributions to the shapes of galaxies!

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For those who'd like to nerd out even more, here's a sketch of Bernoulli's proof that the limit e has to exist and that it has to lie between 2.5 and 3.

(Don't worry if you don't get this math. I promise that you won't need it to make sense of the rest of this thread!)

Bernoulli's proof that the limit $\lim_{N\to\infty}\left(1+\frac{1}{N}\right)^N$ exists and lies between 2.5 and 3

Part 1: For any real $a \in (-1, 0)$ and any integer $m \geq 2$, we have: $(1+a)^m > 1+ma$.

Proof: By induction on m.

Base case: $m = 2$.

$$\text{LHS} = (1+a)^m = (1+a)^2 = 1+2a+a^2$$
$$= 1+ma+a^2 > 1+ma = \text{RHS}.$$

Inductive step: Assume that for any real $a \in (-1,0)$ and any integer $m \in \{2, 3, 4, \dots M\}$, we have:

$$(1+a)^m > 1+ma.$$

What about $m = M+1$?

$$\text{LHS} = (1+a)^m = (1+a)^{M+1} = (1+a)^M (1+a)$$
$$> (1+Ma)(1+a) = 1+(M+1)a + Ma^2$$
$$> 1+(M+1)a = 1+ma = \text{RHS}.$$

$\Rightarrow$ The relation $(1+a)^m > 1+ma$ holds for $m = M+1$ as well.

So, Part 1 is proved.

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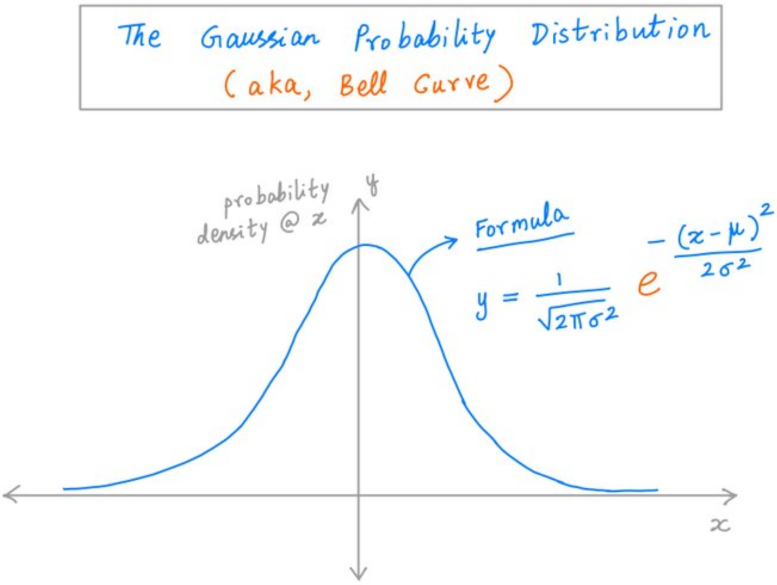
As I said, e plays such a vital role in so many areas beyond capital turns and compounding.

For example, the formula for the Gaussian probability distribution (known more commonly as the "bell curve") depends strongly on e:



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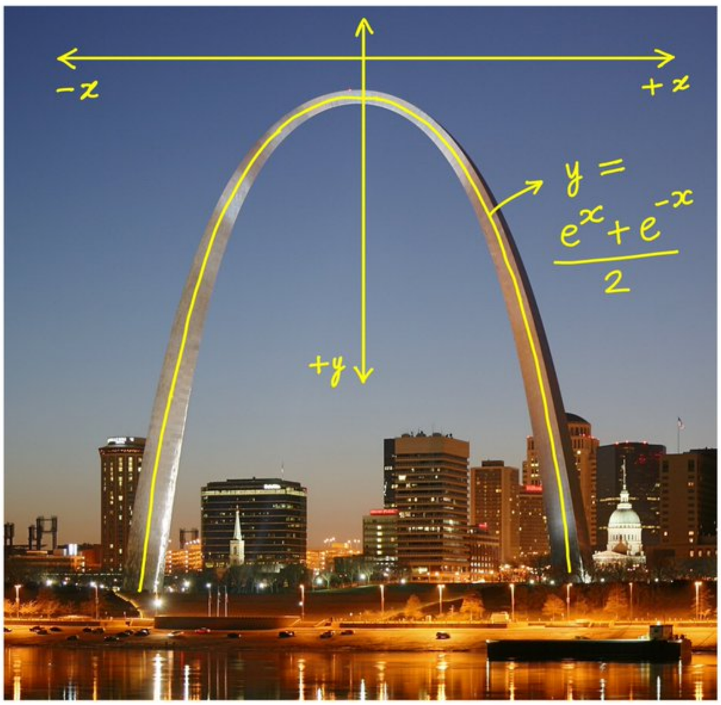
Another fun "curve" whose formula involves e in a big way: the famous Gateway Arch in St. Louis!



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Oh, and what if our bank paid us a more realistic interest rate -- say, R% per year?

Well, *if* we can turn our capital infinitely often, each $1 of ours becomes e^(R/100) dollars in 1 year.

I set R=100 just so the answer simplified to e dollars. But other Rs work too.

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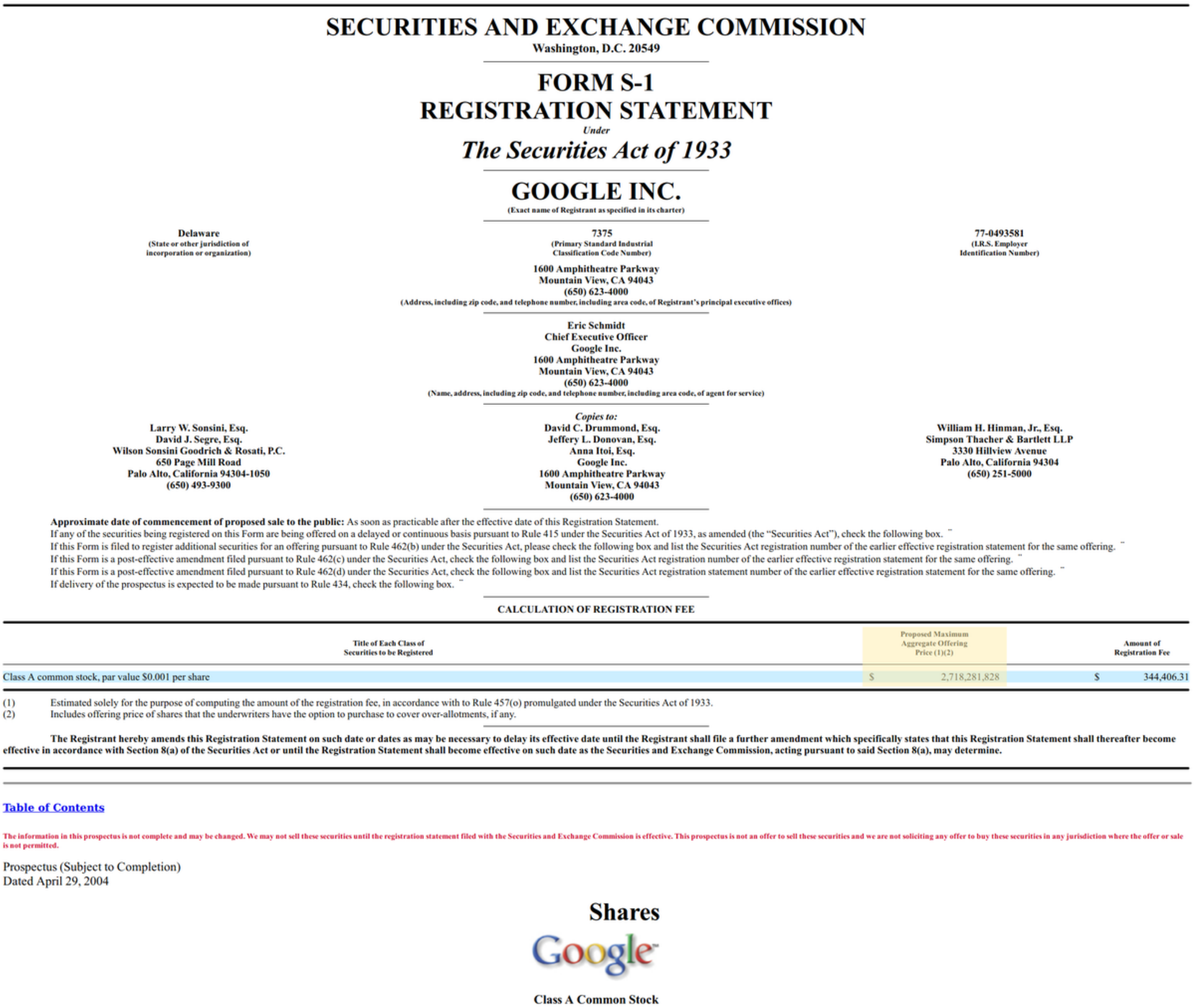
I can't resist pointing out one more place where e showed up unexpectedly: in Google's IPO filing!

https://t.co/If8ikJzU5t

That's e billion dollars, folks!



SECURITIES AND EXCHANGE COMMISSION
Washington, D.C. 20549

FORM S-1
REGISTRATION STATEMENT
Under
The Securities Act of 1933

GOOGLE INC.
(Exact name of Registrant as specified in its charter)

Delaware (State or other jurisdiction of incorporation or organization)	**7375** (Primary Standard Industrial Classification Code Number)	**77-0493581** (I.R.S. Employer Identification Number)

1600 Amphitheatre Parkway
Mountain View, CA 94043
(650) 623-4000
(Address, including zip code, and telephone number, including area code, of Registrant's principal executive offices)

Eric Schmidt
Chief Executive Officer
Google Inc.
1600 Amphitheatre Parkway
Mountain View, CA 94043
(650) 623-4000
(Name, address, including zip code, and telephone number, including area code, of agent for service)

Copies to:

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Approximate date of commencement of proposed sale to the public: As soon as practicable after the effective date of this Registration Statement.
If any of the securities being registered on this Form are being offered on a delayed or continuous basis pursuant to Rule 415 under the Securities Act of 1933, as amended (the "Securities Act"), check the following box. ¨
If this Form is filed to register additional securities for an offering pursuant to Rule 462(b) under the Securities Act, please check the following box and list the Securities Act registration number of the earlier effective registration statement for the same offering. ¨
If this Form is a post-effective amendment filed pursuant to Rule 462(c) under the Securities Act, check the following box and list the Securities Act registration number of the earlier effective registration statement for the same offering. ¨
If this Form is a post-effective amendment filed pursuant to Rule 462(d) under the Securities Act, check the following box and list the Securities Act registration statement number of the earlier effective registration statement for the same offering. ¨
If delivery of the prospectus is expected to be made pursuant to Rule 434, check the following box. ¨

CALCULATION OF REGISTRATION FEE

Title of Each Class of Securities to be Registered	Proposed Maximum Aggregate Offering Price (1)(2)	Amount of Registration Fee
Class A common stock, par value $0.001 per share	$ 2,718,281,828	$ 344,406.31

(1) Estimated solely for the purpose of computing the amount of the registration fee, in accordance with to Rule 457(o) promulgated under the Securities Act of 1933.
(2) Includes offering price of shares that the underwriters have the option to purchase to cover over-allotments, if any.

The Registrant hereby amends this Registration Statement on such date or dates as may be necessary to delay its effective date until the Registrant shall file a further amendment which specifically states that this Registration Statement shall thereafter become effective in accordance with Section 8(a) of the Securities Act or until the Registration Statement shall become effective on such date as the Securities and Exchange Commission, acting pursuant to said Section 8(a), may determine.

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The information in this prospectus is not complete and may be changed. We may not sell these securities until the registration statement filed with the Securities and Exchange Commission is effective. This prospectus is not an offer to sell these securities and we are not soliciting any offer to buy these securities in any jurisdiction where the offer or sale is not permitted.

Prospectus (Subject to Completion)
Dated April 29, 2004

Shares

Google

Class A Common Stock

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If you want to learn more about the rich history behind e, and some of the math surrounding it, I recommend the book "e: The Story of A Number" by Eli Maor. It's a great read!

Amazon link: https://t.co/pVlJE1Rzap

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For more on the investing principles behind capital turns, inventory turns, and working capital management: https://t.co/CeEcrX6JY8

> 1/
>
> Get a cup of coffee.
>
> In this thread, I'll help you understand inventory turnover.
>
> — 10-K Diver (@10kdiver) September 12, 2020

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I also very much enjoyed this Focused Compounding episode, where Andrew and Geoff discuss some of these concepts in their usual illuminating way (h/t @FocusedCompound).

~45 minute video: https://t.co/ZYDmvCOp5n

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If you're still with me, thank you very much!

Capital turns are a key concept in compounding.

In addition to investing insights, I hope this thread also gave you an appreciation for the rich history and the beautiful math behind this topic.

Enjoy your weekend!

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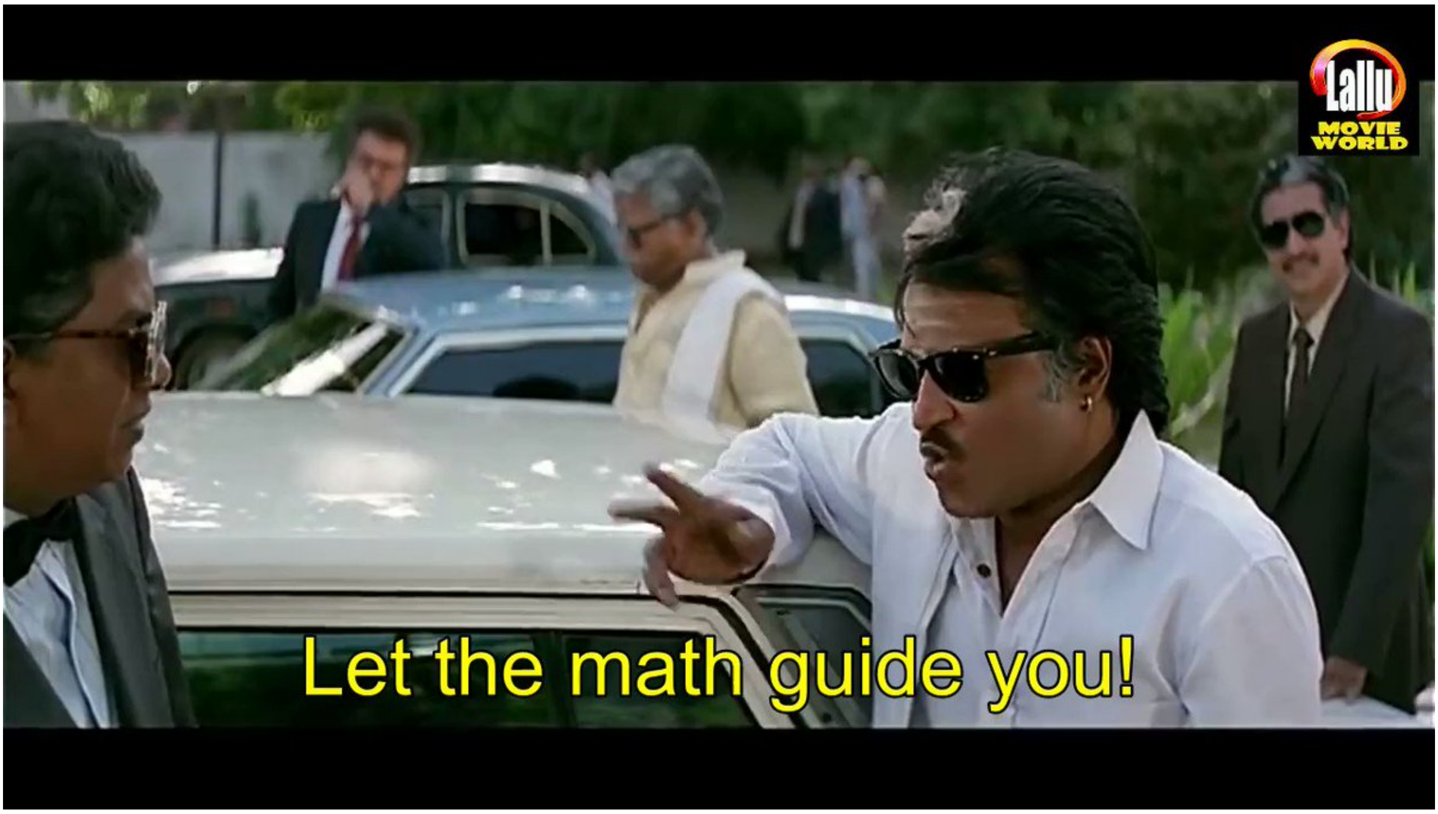
Let the math guide you!